DC Motor Inertia Estimation for Robust Bilateral Control

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***Abstract*— Identification of system parameters of a small DC motor is a complex and challenging task. This research proposes a disturbance observer (DOB) based novel Change of Inertia Observer (CIOB) to estimate the moment of inertia of a DC motor. Moment of inertia of a small DC motor is estimated using CIOB based velocity test and reaction torque observer (RTOB) based inverse motion acceleration test, and the results are compared with conventional acceleration and deceleration motion tests. These two conventional tests are more suitable for large DC motors and less accurate for small DC motors. DOB is used to estimate the disturbance torques and CIOB is to find the change of inertia. Estimated moments of inertia values using the proposed methods are compared with the conventional methods by applying the values to a bilateral teleoperation system. Proposed methods have produced better results than in the conventional methods. Proposed CIOB method of inertia estimation is much simpler and easier to use compared to conventional methods.**

***Keywords*— Bilateral teleoperation, change of inertia observer (CIOB), DC motor parameters, moment of inertia**

I. INTRODUCTION

Advanced motion control plays an important role in modern industrial automation applications. In most of the applications the function of the controller mainly depends on the accuracy of the system parameters [1,2]. For example, in robotic applications, small DC motors are widely used for motion control tasks. Identifying the real DC motor parameters is of great benefit in designing a good motion controller. There are straight forward, well-known techniques available to calculate accurate system parameters such as armature current I_a, armature resistance R_a, armature inductance L_a and torque coefficient K_t etc. However, estimating a precise motor inertia is a challenging task [3,4]. In many DC motor applications, the manufacturer given moment of inertia is considered as the actual inertia value. Sometimes, the manufacturer given value is different from the real motor inertia when different loads such as encoders are connected to the system subsequently. Using a different inertia value may lead to erroneous system response in robotics applications such as medical robotics and aerospace automation missions where precision is important.

Tuning the controller parameters for the desired system response can be achieved only with accurately estimated physical parameters of the system. The desired system response is not achievable if controller tuning is done based only on nominal motor parameters without considering the physical parameter variation [5]. There is no straight forward method to estimate the DC motor mechanical parameters [6-10]. Whei-Min Lin et al. [11] illustrated the importance of estimation of parameters but no method was presented for estimating the inertia and the friction. There are researches to estimate the viscous friction of a DC motor used in precise applications such as position control, but the torque coefficient and motor inertia are not taken into consideration [12-15].

The nominal motor inertia J_n is provided by the manufacturer or initially calculated. However, the actual moment of inertia of the motor may be different from the nominal value due to several reasons; such as addition or removal of accessories to rotary shaft, errors in manufacturer's estimation and wear and tear effects etc. The change of inertia *ǻJ* represents the difference between the nominal and actual inertia values. Estimating *ǻJ* is the main focus in this paper.

In this paper, a novel change of inertia observer concept (an extension of RTOB [16-20]) for motion control is introduced. Proposed CIOB tool is much simple and easy to use compared to conventional methods. Authors propose to use CIOB to find the actual motor inertia of small DC motors. CIOB methodology is explained in the modeling section.

The estimated motor inertia values using CIOB based velocity test, inverse motion acceleration test and conventional acceleration and deceleration tests are applied to a bilateral control system and checked for performance in terms of the position and torque responses. Moment of inertia is one of the depending factors for the stability of the bilateral control system [22].

This paper is organized as follows. In Section 2, DC motor and disturbance observer modeling are explained. Conventional methods to calculate motor inertia are also discussed. Further, in this section, change of inertia observer and inverse motion acceleration test are introduced as new methods to calculate the motor inertia. This is the main contribution of this paper. Then, the estimated moment of inertia values are tested with a bilateral control system. The experimental results are shown in Section 3. Finally, the paper is summarized in Section 4.

II. MODELLING

The following equations (1-4) can be obtained from the electrical representation of a DC motor. Where, E_b - the back emf, T_m - motor torque, K_e - back emf constant, Ȧ - angular speed, K_t - torque coefficient and I_a - armature current.

$$V_a = L\frac{dI_a}{dt} + RI_a + E_b \tag{1}$$

$$E_b = K_e\omega \tag{2}$$

$$T_m = K_t I_a \tag{3}$$

Considering the mechanical parameters of the motor, the motor torque can be written as;

$$T_m = J\frac{d\omega}{dt} + T_f + B\omega + T_l \tag{4}$$

Where, J - motor inertia, T_f - static friction, B - viscous friction coefficient, and T_l - load torque.

The integrated disturbance observer measures and compensates the disturbances to the system and hence it can be effectively used in motion control applications of robotics [3]. The disturbance torque of the motor T_{dis} can be obtained from (5). Here, K_t is the torque coefficient, J is the inertia of the load coupled with the rotor and the subscript n is used to denote the nominal values.

$$T_{dis} = K_{tn}I_a - J_n\dot{\omega} \tag{5}$$

If the nominal motor inertia J_n varies by ΔJ and nominal torque coefficient K_{tn} varies by ΔK_t, the actual J and K_t can be represented by (6) and (7) respectively. Here, nominal torque coefficient and nominal motor inertia values are known from the manufacturer's specifications. Armature current and angular speed can easily be measured using an ammeter and an encoder respectively.

$$J = J_n + \Delta J \tag{6}$$

$$K_t = K_{tn} + \Delta K_t \tag{7}$$

$$T_{dis} = T_l + T_f + B\omega + \Delta J\dot{\omega} - \Delta K_t I_a \tag{8}$$

Disturbance torque T_{dis} in (8) consists of load torque T_l, frictional torque $T_f + B\theta$ and torques arising from parameter variations. T_{dis} can be calculated by using the known parameters of (5). Then, the disturbance observer output is the estimated disturbance torque $\hat{T}_{dis}$ given by (9). Where, G_{dis} is the disturbance gain.

$$\hat{T}_{dis} = \frac{G_{dis}}{(s + G_{dis})}T_{dis} \tag{9}$$

The disturbance observer calculates and estimates the reaction torque as quickly as possible [21]. Using the disturbance feedback it could compensate for the unknown disturbances acting on the system [22]. Furthermore, from the disturbance output, if the frictional components are measured and eliminated, then, the real reaction torque can be measured. This is a variant of the disturbance observer, and it is called the reaction torque observer (RTOB) [19].

A. Conventional inertia estimation methods

The conventional acceleration and deceleration tests [8-10] can be modeled from their native equations as follows:

1) Acceleration motion test.

$$Motor\ Inertia\,(J) = \frac{Acceleration\ Torque}{Acceleration}(kgm^2) \tag{10}$$

2) Deceleration motion test.

This test can be performed as follows. The DC motor is switched off when it is running at its rated speed, and then, the motor speed reduces to zero from its steady speed. The dynamic torque equation for this test is given by (11). The time domain solution for (11) can be expressed as (12). Where, ω_{ss} is the steady state speed.

While the motor speed decreases from steady state speed to zero, following equations (11-13) can be derived to calculate the inertia. Where, ω_{ss} is the steady state speed and τ is mechanical time constant

$$J\dot{\omega} + T_f + B\omega = 0 \tag{11}$$

$$\omega = (\omega_{ss} - \frac{T_f}{J})e^{-(B/J)t} \tag{12}$$

(5)

$$J = B\tau \tag{13}$$

B. Proposed change of inertia observer

Disturbance observer can be used not only for disturbance compensation but also to estimate the change of motor inertia. The disturbance observer is able to estimate the torque variation caused due to the change of the moment of inertia. This estimation is done without using any torque sensor and only by identifying the internal disturbance of the system [18-20].

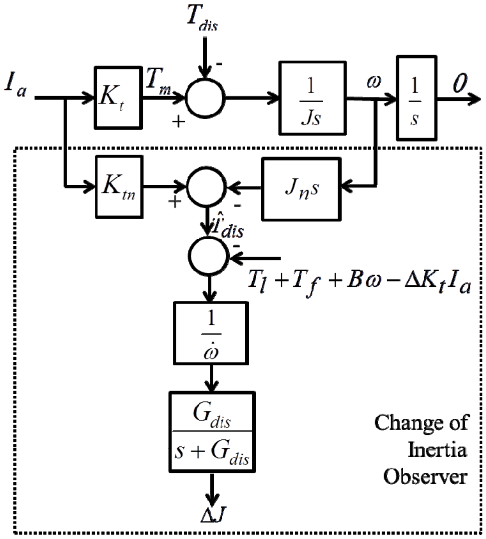


Fig. 1. Change of inertia observer.

Disturbance observer output T_{dis} is calculated using known parameters of (5). A low pass filter with disturbance gain G_{dis} is used to suppress noise components added by differentiation block $J_n s$. Load torque, frictional torque and torque due to motor constant variation are removed from the DOB output. Then, the DOB output consists of only the torque components of the moment of inertia variation. Therefore, the

CIOB output becomes the change of inertia ΔJ .

The moment of inertia of the DC motor is calculated with this novel tool. This is a disturbance observer based sensor used to measure the variation of the motor inertia ΔJ . The total disturbance to the system is given by (8).

The load torque T_l can be made zero by conducting the test in the no load condition. The variation of the motor torque coefficient is usually insignificant or it can be easily calculated [7]. If ΔK_t is considered to be insignificant, then the total disturbance can be expressed by (14).

$$T_{dis} = T_f + B\omega + \Delta J\dot{\omega} \tag{14}$$

The friction components are separately calculated and compensated to the system as shown in Fig. 1. Therefore, the CIOB output represents the estimated change of inertia of the motor ΔJ .This tool can be effectively used to find the real inertia value of the system. The ΔJ variation can be added or subtracted from the nominal motor inertia and it is possible to fine tune the system by analyzing the torque response graphs as shown in Fig. 2 (a) and (b), for positive and negative ΔJ values respectively. The dotted lines in Fig. 2 (a) and (b) represent the torque variation when the applied motor inertia value equals the actual value.

The motor is accelerated from zero to a constant velocity. The acceleration is kept constant over the accelerating period. The torque response of this test can be further elaborated with the aid of Fig. 2. Figure 2 (a) and (b) are graphical explanations for (14). The deviation of the moment of inertia from the nominal moment of inertia is represented by the shaded areas of Fig. 2 (a) and (b). In this proposed CIOB test, ΔJ can be identified from the CIOB output and from the torque versus time plot. Then, by adjusting the nominal moment of inertia by ΔJ , the torque response becomes dependent only on friction components.

C. *Inverse motion acceleration test*

In this approach, RTOB is used as a torque sensor. The calculated values for motor parameters (K_t, T_f and B) are used for this test. A DOB based robust velocity controller is used to achieve accurate velocity responses [23]. The test was conducted by reversing the motor direction with controlled deceleration while it was running at steady state speed. The immediate change of the direction is governed by the controller. As shown in Fig. 3, the motor acceleration to the opposing direction starts at t_1 and ends at t_2. At t_2, the motor comes to a steady state speed of the new direction. This direction variation results in a variation of torque. Motor inertia is directly estimated using the variation of torque.

The RTOB output of the inverse motion acceleration test, $\hat{T}_{dis}$ consists of only the change of motor inertia. The other disturbance components are known and eliminated at the RTOB.

$$\hat{T}_{dis} = \Delta J\dot{\omega} = (J - J_n)\dot{\omega} \tag{15}$$

$$\hat{T}_{dis} = (J - J_n)\dot{\omega} \tag{16}$$

$$\int_{t_1}^{t_2} \hat{T}_{dis}dt = (J - J_n)[\omega(t_2) - \omega(t_1)] \tag{17}$$

$$J = J_n + \frac{1}{[\omega(t_2) - \omega(t_1)]}\int_{t_1}^{t_2} \hat{T}_{dis}dt \tag{18}$$

By combining (8) and (15), $\hat{T}_{dis}$ can be expressed in (16). Then, in (17) the expression is integrated for the time interval t_1 to t_2. Motor inertia J can be calculated from the right hand side known parameters of (18).

In the inverse motion acceleration test, the friction components are compensated for, together with other disturbances. The friction components are calculated by conducting the unidirectional constant velocity test [23]. But in this test, the motor direction is reversed and the region of operation consists of the frictional effects for both directions of the DC motor. Normally, the friction components are different for both motor directions, and friction non-linearity also affects the friction estimation. Therefore, the compensated friction components may not be accurate. The test result of inverse motion acceleration test is included in the results section.

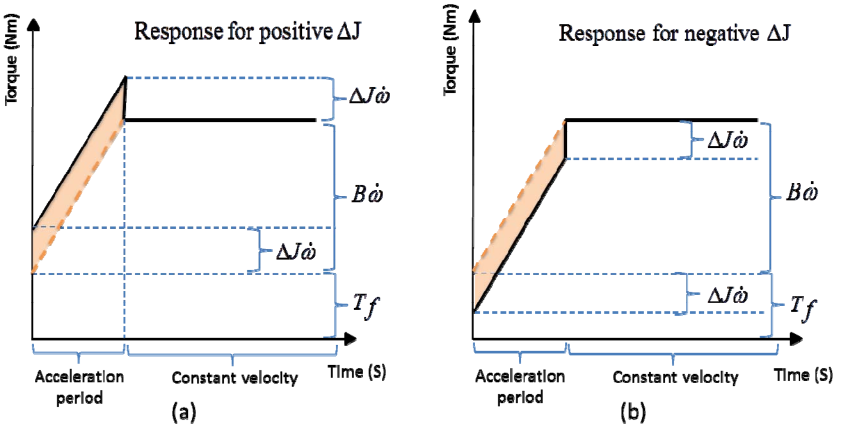


Fig. 2. Torque responses. (a) Torque responses for positive ΔJ . (b) Torque responses for negative ΔJ .

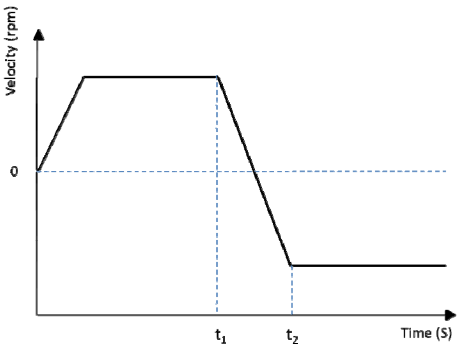


Fig. 3. Velocity response of the inverse motion acceleration test.

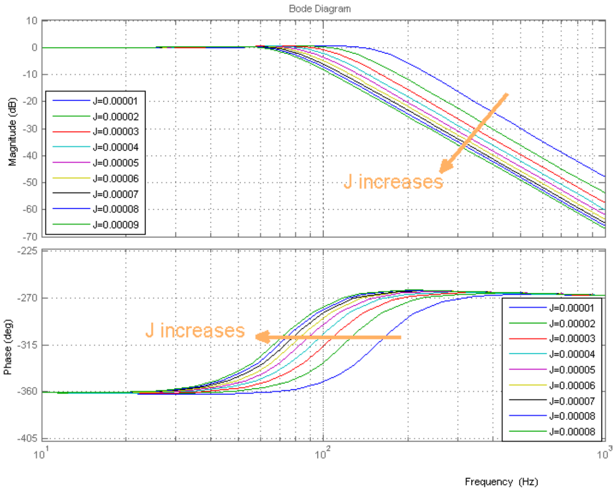


Fig. 4. Frequency response of the master or slave system for position control with different motor inertia values.

III. RESULTS AND DISCUSSION

A. *Selecting bilateral control for inertia verification*

Bilateral control is one of the most widely used teleoperation technologies today [3]. Its controller is based on realization of law of action and reaction between the operator and the environment. It controls the master and the slave sides from the responses of the slave and master sides respectively [3,23]. As far as the operationality improvement of the bilateral control is concerned, identifying the accurate system parameters is an essential task [21-23]. Amongst the system parameters, identifying the actual moment of inertia of the DC motor is not straight forward. When the manufacturer provides a nominal inertia value that does not represent the actual inertia of the motor, the system will produce undesirable responses. In a bilateral control system, one to one position and torque responses are expected. Here, the bilateral teleoperation system is used to verify the accuracy of the inertia values calculated from the four tests discussed in Section II.A-C.

The bilateral teleoperation system used for this experiment consists of two identical modules called master and slave. A module is modeled in the Matlab Simulink environment, and the frequency responses are analyzed for position response by changing the inertia value of the motor from 0.00001 – 0.00009 kgm^2. This simulation was done to identify the effect of the changing motor inertia of the DC motor. According the simulation results in Fig. 4, there is a significant change in the system bandwidth when the moment of inertia is changed within this considered range. Therefore, it is important to identify the exact inertia value of the motor to achieve the desired system response. Inability to identify the correct motor inertia will lead the system to an undesired state.

B. *Experimental setup*

The hardware arrangement of the experiment is shown in Fig. 5. The specifications of the motor are listed in Table I. The motor is driven by a PWM based motor driver with a driver IC (DRV8432 by Texas Instrument) which can carry current up to 14A with 24A peak load. The PWM signals are generated by the processor. An encoder is coupled to the motor to sense the position.

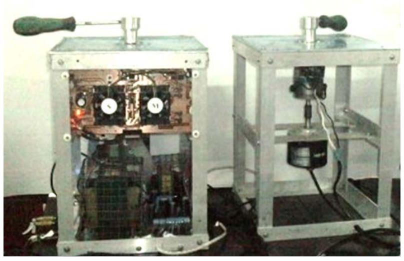


Fig. 5. Bilateral test platform

All computations are written in C language under real time operating system (RTOS) with a sampling time of 100µs. Time critical main control program was invoked in every 100µs, while the data writing task is given the lowest priority. Estimation the motor inertia

TABLE I. DC MOTOR SPECIFICATIONS

Parameter	Value	Unit
Rated output	0.2	kW
Rated/max. torque	20.5/169.5	Ncm
Encoder resolution	2500	Pulses/rev

C. *Estimation of motor inertia experimentally*

The moment of inertia of the DC motor is estimated by using four tests for comparison purposes; conventional acceleration and de-acceleration tests, CIOB based velocity test and reaction torque observer (RTOB) based inverse motion acceleration test.

1) *Acceleration motion test results.*

In this test, the torque is recorded at the controller itself. Equation (10) is used to calculate the motor inertia and the resulting inertia value is 0.000051 kgm^2. The corresponding velocity response of the acceleration motion test is presented in Fig. 6 (a).

2) *Deceleration motion test results.*

Velocity response of deceleration motion test is shown in Fig. 6 (b). The calculated motor inertia in the deceleration motion test by using (13) is 0.000032 kgm^2.

3) *Change of inertia motion test results.*

The velocity and torque responses of CIOB based constant velocity test are shown in Fig. 6 (c) and (d) respectively. The estimated moment of inertia, by using (14) and Fig. 6 (d) is 0.000072 kgm^2. Torque response of Fig. 6 (d) follows the shape of torque response for positive ΔJ of Fig. 2 (a). However, during the acceleration time the torque response is not linear and instead it takes a slightly curved shape. This is due to the low pass filter at DOB, but it does not affect the estimation of ΔJ .

4) *Inverse motion acceleration test results.*

In this test, the motor inertia is calculated from (18). Velocity response of the test is displayed in Fig. 6 (e). The calculated motor inertia is 0.000091 kgm^2.

D. *Validating the results with bilateral teleoperation*

The four motor inertia values calculated from the four tests described in the modeling section are applied to a bilateral control system and tested. Bilateral control system is operated under free and contact motions, and position and torque responses are plotted against time for each inertia value. While doing this series of experiments, all the parameters except motor inertia are kept constant for comparison purposes.

Figure 7 (a)–(h) represent the position and torque responses of the bilateral controller for the four different moments of inertia values. In the bilateral control system, the position response of the slave device should follow that of the master device. The torque response of slave device should be the mirror image of that of the master device in the graphical representation.

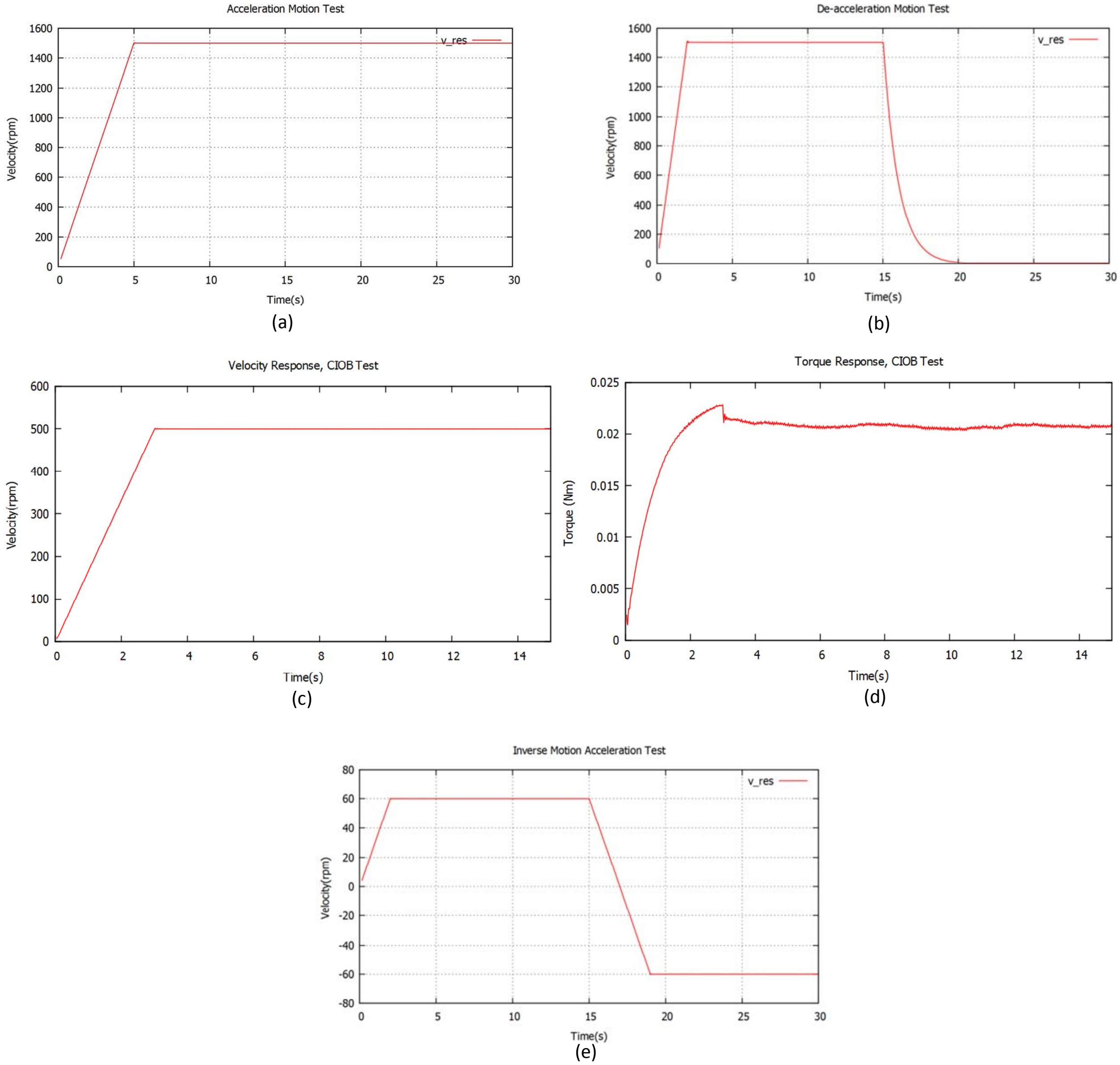
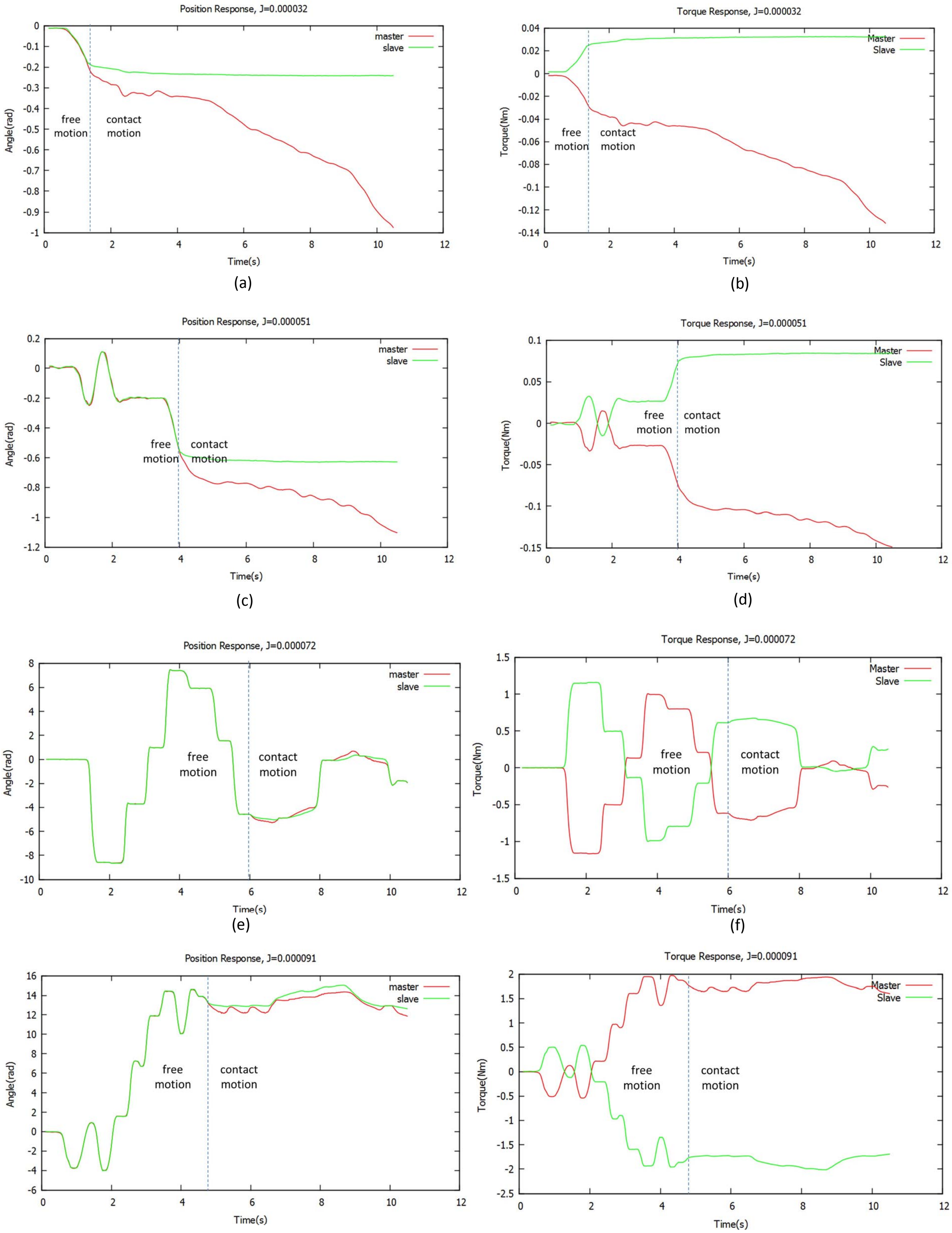


Fig. 6. Velocity responses. (a) Velocity response of acceleration motion test. (b) Velocity response of deceleration motion test. (c) Velocity response of CIOB test. (d) Torque response of CIOB test. (e) Velocity response of the inverse motion acceleration test.



(a) (b) (c) (d) (e) (f) (g) (h)

Fig. 7. Position and torque responses. (a) Position response when the inertia value calculated by acceleration motion test is applied. (b) Torque response when the inertia value calculated by acceleration motion test is applied. (c) Position response when the inertia value calculated by deceleration motion test is applied. (d) Torque response when the inertia value calculated by deceleration motion test is applied. (e) Position response when the inertia value calculated by CIOB test is applied. (f) Torque response when the inertia value calculated by CIOB test is applied. (g) Position response when the inertia value calculated by inverse motion acceleration test is applied. (h) Torque response when the inertia value calculated by inverse motion acceleration test is applied.

The figures corresponding to the deceleration motion test (Fig. 7 (c) and (d)) do not show the desired system performance. Fig. 7 (a) and (b) show improved performances in position and torque responses for the moment of inertia calculated by acceleration motion test compared to the deceleration motion test.

However, the system responses for the moment of inertia values of inverse motion acceleration test and CIOB-test, as shown in Fig. 7 (e)-(h), are satisfactory representations for the bilateral teleoperation. Fig. 7 (e) and (f) show the best performances among these responses. The moment of inertia estimated from CIOB test gives the best performance in the bilateral control system. The position and torque responses of the bilateral control show the minimum errors in this case, and hence the calculated moment of inertia using CIOB test matches to the bilateral control system. Position is the dominant characteristic under free motion. Position responses of Fig. 7 show that the position error is very small for the proposed methods.

IV. CONCLUSION

In this paper, two novel methods of estimating the moment of inertia of a small DC motor are proposed. In this research, no torque sensors are used for the proposed methods. DOB and its variant RTOB are used to attain robustness and to measure the torque respectively. The first method, CIOB is a tool that provides the error of the nominal motor inertia such that it can be used as a tool to derive the actual inertia value. This tool can be used in motion control applications to adjust the moment of inertia parameter to its real value. The second method is inverse motion acceleration test. The estimated moment of inertia by using inverse motion acceleration test includes the friction nonlinearity. The known friction components removed by RTOB considers friction components as linear. Therefore, in this method, the calculated motor inertia is affected by friction nonlinearity. The test results are compared with conventional acceleration and deceleration test outputs. The estimated moment of inertia values of these four tests are separately applied to conventional bilateral control system and the position and torque responses of the bilateral teleoperation were analyzed. The experimental results prove the viability of the proposed methods. These two methods: inverse motion acceleration test and CIOB can be used to estimate the motor inertia of small DC motors and hence improve the robustness of motion control systems.

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